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PARAMETERS OF BINARY-MIXTURE FILM CONDENSATION

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Abstract—As far as the flow of the liquid film is concerned the film condensation of a binary mixture is similar in many aspects to the Nusselt theory. The heat is removed by the film solely. In addition there takes place a diffusional process within the saturated gas flow. It causes the transport of the more volatile component of the mixture in the direction away from the interface. Despite the large number of dimensionless groups only a few parameters are essential for the characterization of the flow. As a consequence processes involving very different binary mixtures may be compared. In the present study laminar flows at a flat plate and between parallel walls are considered only but the discussion of the essential parameters should be of more general importance.

NOMENCLATURE

C,	mass fraction of volatile species;
c_p,	specific heat at constant pressure;
D,	binary diffusion coefficient;
Gr,	Grashof number;
g,	acceleration of gravity;
He,	Stefan number;
h,	enthalpy;
K_l,	nondimensional parameter, equation (38);
k,	thermal conductivity;
L,	length of the cooled wall;
m_i,	molecular weight of species i;
$\dot{m}$,	rate of condensation per unit area;
Pr,	Prandtl number;
p,	pressure;
R,	half width of duct;
Re,	Reynolds number;
Sc,	Schmidt number;
T,	temperature;
U, V,	characteristic velocity components;
u, v,	axial and normal velocity components;
X, Y,	characteristic lengths;
x, y,	axial and normal coordinates.

Greek symbols

β,	Boussinesq factor;
Γ,	dimensionless concentration, equation (42);
δ,	condensate film thickness;
Δ,	$= \{C_e(T) - C_e(T_0)\}/\{C_e(T) - \bar{C}_e(T)\}$, equation (27);
θ,	dimensionless temperature, equation (6);
λ,	latent heat of vaporization;
μ,	dynamic viscosity;
ν,	kinematic viscosity;
ρ,	density.

Subscripts

e,	phase equilibrium;
i,	liquid–vapour interface;
0,	center of duct;
r,	reference value;
w,	wall;
1, 2,	species.

Superscript

$-$,	condensate.

1. INTRODUCTION

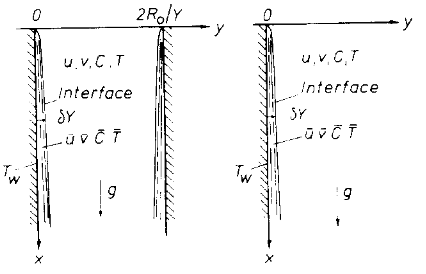
When a saturated vapor is introduced onto a sufficiently cooled wall, the condensate will be formed adjacent to the surface of the wall, flowing downward along the wall under the action of gravity. Usually, the condensate is removed by making use of the gravity flow along vertical walls, being an important device in the chemical engineering field. The phases of the fluid at the liquid–vapor interface may be assumed to be in thermodynamic equilibrium. In the case of one-component vapor, the equilibrium temperature, hence, the temperature at the interface is a function of only the system pressure. The amount of condensation onto the wall is controlled only by the removal process of heat released due to condensation at the interface through the equation of energy.

For binary mixtures, the equilibrium temperature is a function of species concentration as well as the system pressure. The equilibrium concentration in the phases will generally differ for each component. Removal of the volatile component increases or decreases the equilibrium temperature owing to the equilibrium characteristics of the mixture. Due to the concentration difference, the liquid usually contains less of the volatile component than does the vapor. In order to maintain condensation at the interface, the volatile component of the vapor adjacent to the liquid must be removed from the interface. The process of mass transfer controls this removal, whereas the equilibrium temperature, hence, equilibrium concentration will be determined by the energy transfer process. Out of these two processes, which is the rate-controlling process depends mainly upon the equilibrium characteristics. When the equilibrium concentrations are highly sensitive to the temperature, the heat transfer will be the main process controlling the condensation. In most cases, however, the mass transfer may be the rate-controlling factor.

The equilibrium characteristics inherently pertain to the mixture. As such characteristics proper to the system play an important role, its physical behavior tends to be less similar to those of other systems. Even for quite different equilibrium characteristics, however, one can find some local similarities within certain limits of the parameter range. Thus, from the standpoint of application of the results as well as the understanding of the phenomena, it is worthwhile to examine the general behavior on the basis of the similarity concept.

At high speeds of vapor flow, there occur large disturbance waves and ripples at the liquid–vapor interface in addition to the turbulence in the vapor flow itself. Furthermore, liquid droplets may be torn off from the liquid form and also be reentrained into it. At further higher speeds, the characteristic time of the flow becomes of matter in comparison with that of condensation relaxation. In the present study, apart from such highly complicated phenomena, similarity aspects are considered for film condensation of laminar flows of binary vapor mixtures.

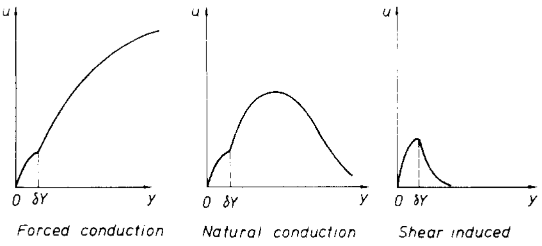
Flow characteristics also are attributive to flow configurations. As for the configuration of vapor flows of film condensation to cooled walls, following systems are considered; (1) forced flows in a ducted wall and on a non-ducted wall, and (2) non-forced flows in a ducted wall and non-ducted wall, Fig. 1. In the latter case, vapor flows will be induced by the natural convection due to density difference or by the shearing effect of the condensate flow. The latter effect will be found to be of little interest. In the present study, for simplicity, the duct is assumed to be of parallel walls. For divergent or convergent walls, similar considerations may also be taken with slight modifications. These flow-configurations show different physical features to be characterized by their own physical scales and properties [1–9], Fig. 2.



Ducted wall

Non-ducted wall (free flow)

FIG. 1. Film flow at ducted and nonducted wall.



Forced conduction

Natural conduction

Shear induced

FIG. 2. Flow configurations.

2. GOVERNING EQUATIONS

A saturated vapor mixture having the temperature T_0 and the mass fraction of the volatile component C_0 is introduced to a cooled wall located vertically, being parallel to the direction of gravitational acceleration, g. Distances from the start of cooling along the wall are measured in terms of the x-coordinate, distances from the wall are measured by the y-coordinate. Concerning the similarity consideration, axisymmetric cylindrical flows are equivalent to two-dimensional flows, hence only two dimensional cases are considered here. The corresponding velocity components in the x- and y-directions are denoted by $u(x,y)$ and $v(x,y)$, respectively. The wall is cooled isothermally at a constant temperature T_w. The vapor condenses onto the surface of the cooled wall and flows downward along it, having the velocity components $\bar{u}$, $\bar{v}$ and forming a thin layer of the thickness $\bar{\delta}$ (x). The superscript $^-$ refers to the condensate.

As the characteristic lengths and velocities in the x- and y-directions of the vapor flow and the condensate flow,

$$X,\ Y;\quad U,\ V;$$
$$\bar{X},\ \bar{Y};\quad \bar{U},\ \bar{V};$$

are chosen and the reference physical properties are denoted by subscript r. As for the x-coordinate, the same scale may be used for the vapor and condensate flows

$$X = \bar{X}. \tag{1}$$

By nondimensionalizing the lengths, velocities and physical properties with their corresponding characteristic or reference values, it will be possible for them to assume values of the order of magnitude unity. The conservation equations of mass, momentum, energy and species will be written in the form of the boundary-layer approximation. In order to avoid unnecessary complications the physical properties of the fluids such as viscosity, conductivity etc. are assumed constant. Of course, the variable properties could also be taken into account, that is, being dependent on the temperature. In such a case the nondimensionalized quantities must be the same function of the nondimensionalized temperature in similar flows. With such assumptions the representation becomes more complicated with less increase in the essential information.

In terms of the nondimensionalized variables the governing equations for the vapor are:

$$\frac{\partial}{\partial x}(\rho u)+\frac{XV}{YU}\frac{\partial}{\partial y}(\rho v)=0; \tag{2}$$

$$\frac{\partial}{\partial x}(\rho uu)+\frac{XV}{YU}\frac{\partial}{\partial x}(\rho vu) = -\frac{p_r}{\rho_r U^2}\frac{\partial p}{\partial x}+\frac{gX}{U^2}\rho+\frac{XV}{YU}\frac{\nu}{VY}\frac{\partial^2 u}{\partial y^2}; \tag{3a}$$

$$p=p(x); \tag{3b}$$

$$\frac{\partial}{\partial x}(\rho u\theta)+\frac{XV}{YU}\frac{\partial}{\partial y}(\rho v\theta) = \frac{XV}{YU}\frac{\nu}{VY}\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{c'_p}{Sc}\frac{\partial C}{\partial y}\frac{\partial\theta}{\partial y}; \tag{4}$$

$$\frac{\partial}{\partial x}(\rho uC)+\frac{XV}{YU}\frac{\partial}{\partial y}(\rho vC)=\frac{XV}{YU}\frac{\nu}{VY}\frac{\rho}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{5}$$

where ρ is the density nondimensionalized by the reference density ρ_r, p the pressure reduced by the reference pressure p_r, ν the kinematic viscosity, c'_p the difference of the specific heats of the species reduced by that of the mixture: $c'_p = (c_{p1} - c_{p2})/c_p$. The Prandtl number Pr and the Schmidt number Sc are respectively

$$Pr=\frac{c_p\mu}{k};\quad Sc=\frac{\nu}{D}$$

where c_p is the specific heat of the mixture at constant pressure, μ the dynamic viscosity, k the heat conductivity and D the binary diffusion coefficient of the volatile component.

The nondimensional temperature

$$\Theta=\frac{T-T_w}{T_0-T_w}=\bar{\Theta} \tag{6}$$

is defined in the same way both in the film and the vapor. The last term of the energy equation, equation (4), represents the energy transport due to the concentration diffusion of species having different specific heats. In the present consideration, it plays no role in the film because heat conduction is predominant and no essential diffusion takes place. In the above equations, the second order effects of thermal diffusion, kinetic energy, viscous and diffusion works and compressible heating are ignored. A system of equations similar to equations (2)–(5) holds for the film flow with the variables $\bar{x}$, $\bar{y}$, $\bar{u}$, $\bar{v}$, $\bar{p}$, $\bar{\theta}$ and $\bar{c}$.

In order to apply the boundary-layer type equations, it is required that

$$(Y/X)^2 \ll 1,\ (\bar{Y}/X)^2 \ll 1 \tag{7}$$

and

$$\rho_r V^2/p_r \ll 1,\ \bar{\rho}_r\bar{V}^2/\bar{p}_r \ll 1. \tag{8}$$

The uniform pressure across the flow passage of vapor and liquid, equation (3b), implies:

$$p_r=\bar{p}_r. \tag{9}$$

Inspection of equation (2) yields

$$\frac{XV}{YU}=1,\ \frac{X\bar{V}}{\bar{Y}\bar{U}}=1 \tag{10}$$

which allow the derivates to be of the order of magnitude of unity.

Although for the right hand side of equation (10) values near unity could also be taken, it would lead only to unnecessary complications. The main conclusion is that the orders of magnitude of the different characteristic scales are not independent. It is simpler to fix them using equation (10).

With these conditions, the governing equations (2)–(5) can be reduced to:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0; \tag{11}$$

$$\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu) = -\frac{p_r}{\rho_r U^2}\frac{\partial p}{\partial x}+\frac{gX}{U^2}\rho+\frac{\nu}{VY}\frac{\partial^2 u}{\partial y^2}, \tag{12}$$

$$\frac{\partial}{\partial x}(\rho u\theta)+\frac{\partial}{\partial y}(\rho v\theta) = \frac{\nu}{VY}\left[\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{c'_p}{Sc}\frac{\partial C}{\partial y}\frac{\partial\theta}{\partial y}\right]; \tag{13}$$

$$\frac{\partial}{\partial x}(\rho uC)+\frac{\partial}{\partial y}(\rho vC)=\frac{\nu}{VY}\frac{\rho}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{14}$$

which now contain the nondimensional parameters of the vapor flow,

$$\frac{VY}{\nu},\frac{gX}{U^2},\frac{\rho_r U^2}{p_r},Pr,\ Sc,\ c'_p=\frac{c_{p1}-c_{p2}}{c_p}. \tag{15}$$

As for the liquid flow a similar group of parameters is considered. Equations (11)–(14) are differential equations of the second order with respect to y and require two relevant boundary conditions.

3. BOUNDARY CONDITIONS

At the cooled wall the no-slip condition of velocity and the temperature is assumed. Usually the wall surface is impermeable to both components of the mixture. These conditions give

$$\bar{u}=0,\ \bar{v}=0,\ \bar{\theta}=0,\frac{\partial\bar{C}}{\partial\bar{y}}=0 \text{ at } \bar{y}=0. \tag{16}$$

On the vapor side, two cases of boundary layer conditions are considered, that of ducted walls and of a non-ducted wall. For a ducted channel with walls situated at $y = 0$ and $y = 2R_0/Y$, where $2R_0$ is the distance of the walls:

$$\frac{\partial u}{\partial y} = 0, \frac{\partial \theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 \text{ at } y = R_0/Y. \qquad (17a)$$

In the case of non-ducted walls, one can specify the vapor conditions at infinity as

$$u = U_0, \theta = 1, C = C_0 \text{ at } y \to \infty. \qquad (17b)$$

At the liquid–vapor interface, no transition layer from the vapor phase to the liquid phase is assumed. The interface between the liquid and the vapor has an infinitesimal thickness. The location of the interface is expressed as

$$y = \delta(x), \bar{y} = \bar{\delta}(x). \qquad (18)$$

It should be noted that $\delta(x) \neq \bar{\delta}(x)$, since the film thickness in the liquid and in the vapor is non-dimensionalized by the different characteristic lengths $\bar{Y}$ and Y, respectively. At the interface, the continuity condition of u-velocity and temperature gives

$$Uu_i = \bar{U}\bar{u}_i, \qquad (19)$$

$$\theta_i = \bar{\theta}_i, \qquad (20)$$

where subscript i refers to the liquid–vapor interface.

The transport fluxes of mass, momentum, energy and species from the vapor to the interface have to be equal to these passing from the interface to the liquid plus additional contributions due to sources at the interface.

Assuming in accordance to the boundary–layer condition equation (7) that

$$\left(\frac{Y}{X}\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)^2 \ll 1, \left(\frac{\bar{Y}}{X}\frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x}\right)^2 \ll 1, \qquad (21)$$

the components of the unit vector normal to the interface can be approximated by:

$$\mathbf{n}: -\frac{Y}{X}\frac{\mathrm{d}\delta}{\mathrm{d}x}, +1.$$

The mass flow components of the vapor are:

$$\rho u \rho_r U, \rho v \rho_r V.$$

The mass of vapor condensing at the interface per unit area and unit time, the so-called condensation rate $\dot{m}$, is equal to the negative normal component of the mass flow.

$$\dot{m} = \rho u \rho_r U \frac{Y}{X}\frac{\mathrm{d}\delta}{\mathrm{d}x} - \rho v \rho_r V.$$

The same consideration can be taken for the liquid. With equation (10) the following relation for the condensation rate at the interface is obtained:

$$\dot{m} = \rho_r V \rho_i\left(u_i \frac{\mathrm{d}\delta}{\mathrm{d}x} - v_i\right) = \bar{\rho}_r \bar{V}\left(\bar{u}_i \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x} - \bar{v}_i\right), \qquad (22)$$

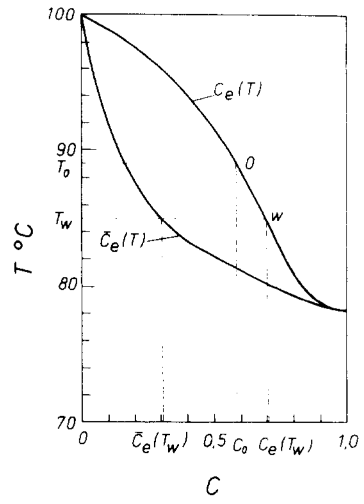


FIG. 3. Phase equilibrium diagram for ethanol–water mixture at atmospheric pressure.

where the constant density of the liquid is assumed, $\bar{\rho} = 1$.

The condition that the frictional forces at the interface have to be continuous gives

$$\mu \frac{U}{Y}\left(\frac{\partial u}{\partial y}\right)_i = \bar{\mu}\frac{\bar{U}}{\bar{Y}}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_i. \qquad (23)$$

The heat released by condensation at the interface must be carried away by heat conduction into both phases. The heat conductivity of the liquid is one order of magnitude greater than that of the vapor. Estimates show that the former will not be compensated by a greater temperature gradient in the vapor. Therefore the heat transport takes place almost in the film only. By denoting the latent heat of condensation of the mixture in the liquid as λ the boundary conditions for the heat transport at the interface can be given by

$$\frac{k}{\bar{Y}}\left(\frac{\partial \bar{\theta}}{\partial \bar{y}}\right)_i = \frac{\lambda}{T_0 - T_w}\dot{m}. \qquad (24)$$

When as in the case of Fig. 3, the concentration of the volatile component $\bar{C}_i$ in the liquid at a certain temperature is smaller than the concentration C_i in the vapor, an apparent source of the volatile species arises at the interface. The difference $(C_i - \bar{C}_i)$ has to be carried away by diffusion. The diffusion in the liquid can be neglected in comparison with the diffusion in the vapor.

This leads to the following boundary condition for the concentration:

$$\dot{m}(C_i - \bar{C}_i) = -\rho_r \rho_i D \frac{1}{Y}\left(\frac{\partial C}{\partial y}\right)_i. \qquad (25)$$

Equations (19), (20) and (22)–(25) give the boundary conditions at the interface.

4. PHASE EQUILIBRIUM

At the liquid–vapor interface, the mixture is assumed to be in phase equilibrium. An example of a phase equilibrium diagram is shown in Fig. 3 for ethanol–water mixture at atmospheric pressure. The pressure changes in the vapor and therefore in the two-phase flow are practically of no significance in comparison with the temperature changes. The equilibrium diagram, Fig. 3 can therefore be taken for a constant pressure, i.e. $p = p_0$. The upper curve gives the equilibrium concentrations of the vapor phase, being denoted by $C_e(T)$, and the lower curve is that of the liquid phase, $\bar{C}_e(T)$. The liquid phase contains less of the volatile component than does the vapor phase; $\bar{C}_e(T) < C_e(T)$. Since no-transition layer between the liquid and vapor phases is taken into account, the mixtures cannot take concentrations between $C_e(T)$ and $\bar{C}_e(T)$, but only values of $C_e(T)$ and $\bar{C}_e(T)$;

$$C_i = C_e(T_i), \bar{C}_i = \bar{C}_e(T_i). \tag{26}$$

Thus, the vapor adjacent to the liquid must be stripped of the volatile component from the interface by the amount of $\dot{m}[\bar{C}_e(T_i) - C_e(T_i)]$. The removal of the volatile component is carried out by the diffusion process, equation (25). The latter may be motivated by the driving force of concentration difference between the interface and the bulk, $[C_e(T_i) - C_0]$. The condensation rate should then be correlated with the ratio of these concentration differences, which is an important factor to characterize the condensation process. The factor is denoted by

$$\Delta = \frac{C_e(T) - C_0}{C_e(T) - \bar{C}_e(T)},$$

$$\Delta_w = \frac{C_e(T_w) - C_0}{C_e(T_w) - \bar{C}_e(T_w)}. \tag{27}$$

The denominator means a barrier for condensation from the vapor to the liquid and the numerator implies a potential of the condensation.

5. CONDENSATE FLOW

Due to small density of the vapor compared with that of the liquid, the condensate layer has usually very thin thickness and small v-velocities. If the Reynolds number based on these properties is assumed sufficiently small the conditions of creeping flow are valid:

$$\frac{\bar{V}\bar{Y}}{\bar{\nu}} \ll 1, \bar{Pr}\frac{\bar{V}\bar{Y}}{\bar{\nu}} \ll 1, \bar{Sc}\frac{\bar{V}\bar{Y}}{\bar{\nu}} \ll 1. \tag{28a}$$

Hence the effect of convection terms in the governing equations of condensate flow can be neglected. This will be confirmed for the different cases later.

Of equation (3a), only terms on the right hand side remain. The pressure term is very small in comparison with the gravity term, because the pressure gradient is the same as in the vapor. In non-ducted flows the pressure is practically constant and in ducted flows the pressure changes are of the order of magnitude of the dynamic pressure, that is

$$p_r \frac{\partial p}{\partial x} \sim \rho_r U^2.$$

The ratio of the first term to the second on the right hand side of equation (3) for ducted flows is then approximately:

$$\frac{p_r}{\bar{\rho}_r \bar{U}^2}\frac{\partial p}{\partial x} \bigg/ \frac{gX}{\bar{U}^2} \sim \frac{\rho_r}{\bar{\rho}_r}\frac{U^2}{gX} \tag{28b}$$

which can be neglected in the present cases. For higher vapor velocities or in the case of divergent or convergent ducted walls, this estimation has to be re-examined. Within the approximations made in this paper it is not necessary to take into account the pressure term in the film flow.

Thus equations (3) to (5) for the liquid are reduced to:

$$\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = -\frac{g\bar{Y}}{\bar{U}\bar{\nu}}; \tag{29a}$$

$$\frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} = -\frac{\bar{Pr}}{\bar{Sc}} c'_p \frac{\partial \bar{C}}{\partial \bar{y}}\frac{\partial \bar{\theta}}{\partial \bar{y}}; \tag{29b}$$

$$\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} = 0, \tag{29c}$$

which are subject to the boundary conditions at the wall and at the liquid–vapor interface:

$$\bar{y} = 0: \bar{u} = 0, \bar{\theta} = 0, \frac{\partial \bar{C}}{\partial \bar{y}} = 0;$$

$$\bar{y} = \bar{\delta}(x): \bar{u} = \bar{u}_i, \bar{\theta} = \bar{\theta}_i, \bar{C} = \bar{C}_i = \bar{C}_e(T_i). \tag{30}$$

Equations (29) and (30) yield:

$$\bar{u} = \bar{u}_i \frac{\bar{y}}{\bar{\delta}} - \frac{1}{2}\frac{g\bar{Y}^2}{\bar{U}\bar{\nu}}(\bar{y}^2 - \bar{\delta}\bar{y}); \tag{31a}$$

$$\bar{\theta} = \theta_i \frac{\bar{y}}{\bar{\delta}}; \tag{31b}$$

$$\bar{C} = \bar{C}_i. \tag{31c}$$

By these solutions, the boundary conditions at the interface, equations (23) to (25) are written as

$$\bar{u}_i - \frac{1}{2}\frac{gY^2}{\bar{U}\bar{\nu}}\bar{\delta}^2 = \frac{\mu}{\bar{\mu}}\frac{U}{\bar{U}}\frac{\bar{Y}}{Y}\bar{\delta}\left(\frac{\partial u}{\partial y}\right)_i; \tag{32a}$$

$$\frac{\bar{k}}{\bar{Y}}\frac{\theta_i}{\bar{\delta}} = \frac{\lambda}{T_0 - T_w}\dot{m}; \tag{32b}$$

$$\dot{m}(\bar{C}_i - C_i) = \frac{\rho_r \rho D}{Y}\left(\frac{\partial C}{\partial y}\right)_i. \tag{32c}$$

In equation (32a) the gravity term is the most predominant in comparison with the right hand side for usual cases. Considering the order of magnitude for the terms of equation (31a) leads to

$$\bar{U} = \frac{g\bar{Y}^2}{\bar{\nu}}, \tag{33}$$

which combined with equation (10) gives:

$$\bar{V} = \frac{g}{\bar{\nu}}\frac{\bar{Y}^3}{X}. \tag{34}$$

Equations (33) and (34) correlate the characteristic velocities $(\bar{U}, \bar{V})$ in the film to the characteristic lengths $(X, \bar{Y})$.

Integrating the equation of continuity for the liquid, equation (11) gives a useful relation for the y-component velocity.

$$\bar{u}_i \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x} - \bar{v}_i = \frac{\mathrm{d}}{\mathrm{d}x} \int_0^{\bar{\delta}} \bar{u}\, \mathrm{d}\bar{y}.$$

With the help of equation (31a) and (33), this gives:

$$\bar{u}_i \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x} - \bar{v}_i = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\bar{\delta}}{2}\left(\bar{u}_i + \frac{1}{6}\bar{\delta}^2\right)\right]. \tag{35}$$

An insight into the film flow is now possible. In usual cases, the shear stress of the vapor at the interface, the right hand side of equation (32a) is very small compared with that of the liquid, that is,

$$\frac{\mu}{\bar{\mu}} \frac{U}{\bar{U}} \frac{\bar{Y}}{Y} \ll 1, \tag{36}$$

which will be checked later, when the vapor flow is more specified. By using equations (33) and (36), equations (32a) and (35) give the x and y component velocities of the liquid at the interface, equation (37a) and (37b) respectively. Further with equations (37a) and (37b), equations (22) and (32b) lead to equations (37c) and (37d) which give the condensation rate and the interfacial temperature, respectively.

$$\bar{u}_i = \frac{1}{2}\bar{\delta}^2; \tag{37a}$$

$$\bar{v} = -\frac{1}{2}\bar{\delta}^2 \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x}; \tag{37b}$$

$$\frac{\dot{m}}{\bar{\rho}_r \bar{V}} = \bar{\delta}^2 \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x}; \tag{37c}$$

$$\theta_i = \frac{\bar{Pr}}{\bar{He}} K_l \bar{\delta}^3 \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x} \tag{37d}$$

where $\bar{He}$ is the Stefan number defined by

$$\bar{He} = \frac{\bar{c}_p(T_0 - T_w)}{\lambda}$$

and K_l is a characteristic parameter for the liquid defined by

$$K_l \equiv \frac{\bar{V}\,\bar{Y}}{\bar{v}} = \frac{g\bar{Y}^4}{\bar{v}^2 X}. \tag{38a}$$

In the case of one-component condensation, the interfacial temperature can be regarded as one of the specified quantities of the system namely $\theta_i = 1$. In equation (37d) one can then put

$$K_l = \frac{\bar{He}}{\bar{Pr}} \tag{38b}$$

and solve equation (37d) to obtain all the aspect of condensate flow – the Nusselt solution. On the other hand, for binary mixtures, the controlling rate is how much of the excessive vapor species can be removed from the interface between the vapor and liquid phases where the mixture is subjected to different equilibrium concentrations. Thus, the vapor flow governs the condensation process mainly through its behavior of mass transfer, as seen later. Nevertheless, it is convenient to introduce the parameter K_l for the purpose of estimating the interfacial temperature as well as viewing a correlation between one and two component systems. By further introducing a reference temperature θ_r at the interface so that $\theta_r \approx \theta_i$, equation (37d) is written as

$$\frac{\theta_i}{\theta r} = \frac{\bar{Pr}}{\bar{He}} \frac{K_l}{\theta_r} \bar{\delta}^3 \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x}.$$

The reference temperature at the interface is then given by

$$\theta_r = \frac{\bar{Pr}}{\bar{He}} K_l. \tag{39}$$

With the parameter K_l, the characteristic scales of the condensate flow can be expressed as

$$\bar{Y} = K_l^{1/4}\left(\frac{\bar{v}^2 X}{g}\right)^{1/4}, \quad \bar{U} = K_l^{1/2}\sqrt{(gx)}, \quad \bar{V} = K_l^{3/4}\left(\frac{\bar{v}^2 g}{X}\right)^{1/4} \tag{40}$$

where K_l is given by the energy transfer relation, equation (38b), for the one-component system and by the mass transfer relation, equation (59), for the two-component system.

The characteristic values given by equation (40) depend on the length of the film X and on the reference temperature at the interface θ_r. For the saturated one-component flows $\theta_r = 1$, whereas for binary mixtures the reference quantities of the film, $\bar{Y}$, $\bar{U}$, $\bar{V}$ are smaller than these for one-component flows of the equivalent fluid under the same boundary conditions.

For the creeping flow conditions (28a) one obtains from equations (38) and (39):

$$\frac{\bar{V}\bar{Y}}{\bar{v}} = K_l; \bar{Pr}\frac{\bar{V}\bar{Y}}{\bar{v}} = K_l \bar{Pr} = \bar{He}, \theta_r;$$

$$\bar{Sc}\frac{\bar{V}\bar{Y}}{\bar{v}} = \bar{Sc}\, K_l = \frac{\bar{Sc}}{\bar{Pr}} \bar{He}\, \theta_r. \tag{41}$$

As an example assume the properties of water $T_0 = 393°$, $\bar{c}_p = 1\,\mathrm{cal\,g^{-1}\,grad}$, $\lambda = 500\,\mathrm{cal\,g^{-1}}$ and cooling of $T_0 - T_w = 25°$, then one gets $\bar{He} = 0.05$. The Prandtl number at this temperature is relatively small $\bar{Pr} = 2$. The film parameter for the one-component flow $(\theta_r = 1)$ is then $K_l = 0.025$. Thus, the creeping flow conditions are satisfied for $\theta_r = 1$, hence more sufficiently for binary mixtures $\theta_r < 1$. Only the third condition (28) may cause difficulties for higher vapor velocities, see equation (60) and following, with respect to the high Schmidt numbers of liquids, $(\bar{Sc}/\bar{Pr}$

$\sim 300/2 = 150$). But even if the third creeping condition (28a) and equation (31c) are not fully satisfied, the concentration gradient in the film is small in any case and can be neglected in the boundary condition (32c) and consequently drops out of consideration.

Assuming a reasonable value of $\bar{v} = 0.3 \times 10^{-6}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$ (f.e. water at 100°C) and a length of $X = 0.10$ m equation (40) gives for the one-component flow of water $\bar{Y} = 0.7 \times 10^{-4}$ m and $\bar{U} = 0,16\,\mathrm{m\,s}^{-1}$. As already known from Nusselt theory the liquid film is very thin and quite slow. This is important for the boundary conditions of the vapor flow.

6. VAPOR FLOW

For the vapor flow, the condition of small Reynolds numbers such as equation (28a) is too restrictive to obtain the general feature of the problem. Usually it is necessary to solve the full set of governing equations, equations (11)–(14).

The natural convection term in equation (12), $\rho g X/U^2$, can be attributed mainly to the local variation of density due to temperature and concentration differences. When the variation is not too large, the effect is expressed by expanding the density about the reference state with respect to temperature and concentration according to Boussinesq's approximation:

$$\rho = 1 + \left(\frac{\partial \rho}{\partial \theta}\right)_{p,C,r}(\theta - \theta_r) + \left(\frac{\partial \rho}{\partial C}\right)_{p,\theta,r}(C - C_r) + \ldots.$$

For low Mach number flows, the influence of the pressure change in the gravitational term can be neglected in comparison to that of the density. From the equation of state of an ideal gas the following relation can be easily found

$$\left(\frac{\partial \rho}{\partial \theta}\right)_{p,C,r} = -\frac{T_0 - T_w}{T_r}.$$

The molecular weight of the mixture of two perfect gases is given by the relationship

$$\frac{1}{m} = \frac{1 - C}{m_1} + \frac{C}{m_2},$$

where m_2 is the molecular weight of the volatile component.

This gives

$$\left(\frac{\partial \rho}{\partial C}\right)_{r_1,\theta,r} = -\frac{m_1 - m_2}{m_1 m_2}, m_r.$$

where m_r is the molecular weight for the reference concentration $C = C_r$.

It is convenient to introduce for the concentration a similar variable as for the temperature, equation (6):

$$\Gamma = \frac{C - C_e(T_w)}{C_0 - C_e(T_w)} \quad \frac{C_e(T_w) - C}{C_e(T_w) - C_0}, \tag{42}$$

which takes values between 0 (at the wall) and 1 (in the free stream). Substituting these expressions of $\partial\rho/\partial\theta$ and $\partial\rho/\partial C$ into the expansion of ρ with $T_r = T_0$ ($\theta_0 = 1$) and $C_r = C_0$ ($\Gamma_0 = 1$) gives

$$\begin{aligned} \rho - 1 &= \frac{T_0 - T_w}{T_0}(1 - \theta) + \frac{m_2 - m_1}{m_1 m_2} m_0[C_e(T_w) - C_0](1 - \Gamma) + \ldots \\ &= \beta_t(1 - \theta) + \beta_c(1 - \Gamma) + \ldots; \end{aligned} \tag{43}$$

$$\beta_t = \frac{T_0 - T_w}{T_0}; \beta_c = \frac{m_2 - m_1}{m_1 m_2} m_0[C_e(T_w) - C_0].$$

The Boussinesq factor for the concentration, β_c, takes under certain conditions (Table 1) values of the order of magnitude unity. In such cases considerable changes of the vapor density will be associated with this and β_c becomes a significant parameter of the flow.

The static pressure gradient in equation (12) is given by the relation:

$$\frac{p_r}{\rho_r}\frac{\partial p_s}{\partial x} = gX.$$

If the pressure of the vapor is measured as the difference from this static pressure caused by gravity, p_s, namely $p' = p - p_s$, the equation of motion equation (12) is written as

$$\frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho v u) = -\frac{p_r}{\rho_r U^2}\frac{\partial p^1}{\partial x} + \frac{gX}{U^2}(\rho - 1) + \frac{v}{VY}\frac{\partial^2 u}{\partial y^2}.$$

In this equation of boundary-layer type, the convective terms and the friction term are to be of the same order of magnitude, that is,

$$\frac{VY}{v} = 1. \tag{44}$$

By introducing the new variables Γ and p' with equations (43) and (44), the system of equations (11)–(14) for the vapor flow is reduced to

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0; \tag{45a}$$

$$\frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho u v) = -\frac{Pr}{\rho_v U^2}\frac{\partial p^1}{\partial x} + \beta_t \frac{gX}{U^2}(1 - \theta) + \beta_c \frac{gX}{U_2}(1 - \Gamma) + \frac{\partial^2 u}{\partial y^2}; \tag{45b}$$

$$\frac{\partial}{\partial x}(\rho u \theta) + \frac{\partial}{\partial x}(\rho v \theta) = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{c_p^f}{Sc}[C_e(T_w) - C_e(T_0)] \times \rho \frac{\partial \Gamma}{\partial y}\frac{\partial \theta}{\partial y}; \tag{45c}$$

$$\frac{\partial}{\partial x}(\rho u\Gamma)+\frac{\partial}{\partial y}(\rho v\Gamma)=\frac{\rho}{Sc}\frac{\partial^2\Gamma}{\partial y^2}. \tag{45d}$$

Additionally it should be noted that the density is given by equation (43).

The boundary conditions at $y = R_0/Y$ or $y = \infty$, equation (17a) or (17b), can easily be written in terms of the reduced quantities. Before formulating the boundary conditions for the vapor at the interface, the condition which specifies the type of flow, namely of forced convection, natural convection and of shear induced flow will be considered.

Natural convection has to be of significance, if the relevant terms in equation (45b) are of the same order of magnitude that is,

$$\beta\frac{gX}{U^2}=1. \tag{46}$$

In this equation, β can be replaced by the Boussinesq factor β_t for the predominant influence of the temperature or by β_c for that of the concentration, equation (43).

For a binary mixture of water–ethanol ($m_1 = 18$, $m_2 = 46$) at atmospheric pressure and a saturation temperature of $T_0 - 273 = 95°\mathrm{C}$, one gets $C_0 = C_e(T_0) = 0.35$, $m_0 = 28.8$. This gives the values of the factor for various wall-temperatures shown in Table 1.

It is seen from Table 1 that binary mixtures of species having a great difference of molecular weight take considerable large values of β_c. According to equation (46), the characteristic reference velocity for natural convection in the vapor is given by

$$U=\sqrt{(\beta gX)}, \tag{47}$$

where β is the largest one between β_t and β_c. Vapor flows induced by the shear stress of the condensate flow are possible only if it overwhelms natural convection, that is, $\bar{U} \gg \sqrt{(\beta gX)}$. In view of the second equation (40), this means

$$\beta \ll K_t, \tag{48}$$

which is rarely realistic. Therefore, the case of shear-induced flows can be excluded from the present consideration, as mentioned in the introduction.

In usual cases of forced flow or natural convection, one can find $\bar{U} \ll U$. The vapor velocity is then assumed to vanish at the interface. For forced flows, the characteristic velocity of the vapor is chosen as

$$U=U_0 \tag{49}$$

where U_0 is the specified velocity at the inlet of ducted walls or the free stream velocity of the flow along the plate. Thus, pure forced convection can be assumed only if:

$$K_\beta=\frac{\sqrt{(\beta gX)}}{U_0}=\sqrt{\left(\frac{\beta gX}{U_0^2}\right)}\ll 1. \tag{50}$$

For small values of β, $K_\beta \ll 1$, that is, natural convection in the vapor can be neglected. If K_β is of order of magnitude unity, natural convection becomes comparable with the forced flow. In this case, it may be significant whether the vapor flow is directed to or against the direction of gravity.

In the following, for the vapor flow specified, two cases will be considered; (a) forced flow, $K_\beta \ll 1$, and (b) natural convection flow, $K_\beta \gg 1$. In each case, two types of the flow configuration are considered; (1) ducted flow and (2) non-ducted free flows. For ducted flows, the characteristic length Y can be taken as the magnitude of R_0; (1) $Y = R_0$. For free flows along a plate the order of magnitude of X can be considered the length of the plate L; (2) $X = L$. The combined configurations of vapor flow are listed in Table 2.

If V is eliminated from equations (10) and (44) one gets:

$$\frac{Y^2}{X}=\frac{\nu}{U}. \tag{51}$$

With these relations in Table 2 and of equation (51), the characteristic quantities, X, Y, U can be easily calculated as shown in Table 3.

In the case of forced flows the Reynolds number

$$Re_R = U_0R_0/\nu \text{ or } Re_L = U_0L/\nu$$

appears in the expression of unspecified characteristic lengths. For natural convection the Grashof number

$$Gr_R = \beta gR_0^3/\nu^2 \text{ or } Gr_L = \beta gL^3/\nu^2$$

appears. It may be practically of advantage to introduce the Grashof number in the relation for U in 1b.

For ducted flows, a certain starting length X_L exists over which the boundary layer on the walls grows to occupy half the duct width. In contrast to free flows along a plate, the velocity at the center increases axially owing to the displacement effect of the boundary layers. Nevertheless X_L can be estimated quite well by putting Y equal for the ducted and for the free flow. With $L = X_L$ one obtains, for forced and natural convection flows respectively,

$$\text{(a1)}\quad \frac{X_L}{R_0}=\frac{U_0R_0}{\nu};\qquad \text{(b1)}\quad \frac{X_L}{R_0}=\frac{\beta gR_0^3}{\nu^2}. \tag{52}$$

In the present case, both the Reynolds number and the Grashof number based on R_0 are large. This means that the flow in the starting length X_L is of importance for ducted flows. At the same time it means that the condition (7) for the vapor is satisfied already at the starting length X_L, and hence is better for the region in greater lengths of the duct.

For free flows, one gets from Table 3:

$$\text{(a2)}\quad \frac{Y}{L}=\left(\frac{\nu}{U_0L}\right)^{1/2};\qquad \text{(b2)}\quad \frac{Y}{L}=\left(\frac{\nu^2}{\beta gL^3}\right)^{1/4} \tag{53}$$

Table 1. Boussinesq factors and Δ_w, equation (27), for water–ethanol mixture

$T_w - 273$	$C_e(T_w)$	$[C_e(T_w) - C_0]$	β_c	β_t	$\bar{C}_e(T_w)$	$[C_e(T_w) - \bar{C}_e(T_w)]$	Δ_w
95	0.35	0	0	0	0.06	0.29	0
90	0.56	0.21	0.200	0.014	0.14	0.42	0.50
85	0.70	0.35	0.340	0.027	0.31	0.39	0.90
80	0.83	0.48	0.420	0.041	0.71	0.12	4.00

Table 2. Vapor-flow configurations

		(a) Forced $U = U_0$	(b) Natural $U = \sqrt{(\beta g X)}$
(1) Ducted	$Y = R_0$	a_1	b_1
(2) Free	$X = L$	a_2	b_2

Table 3. Characteristic quantities of the vapor flow

	(a) Forced			(b) Natural		
	X	Y	U	X	Y	U
(1) Ducted	$R_0 Re_R$	R_0	U_0	$R_0 Gr_R$	R_0	$\sqrt{(\beta g R_0)} Gr_R^{1/2}$
(2) Free	L	$L Re_L^{-1/2}$	U^0	L	$L Gr_L^{-1/4}$	$\sqrt{(\beta g L)}$

where the Reynolds number and Grashof number are based on L. Assuming that $v = 0.2 \times 10^{-4}\,\mathrm{m^2\,s^{-1}}$, $U_0 = 10\,\mathrm{m\,s^{-1}}$, $L = 0.10\,\mathrm{m}$, $\beta = 0.20\,\mathrm{m}$, $g = 10\,\mathrm{m\,s^{-2}}$, one gets for the two cases, respectively

$$\text{(a2)}\quad Y/L = 0.50 \times 10^{-2}; \quad \text{(b2)}\quad Y/L = 2 \times 10^{-2}.$$

Also in this case the condition (7) for the boundary-layer type flow is satisfied.

A comparison of Y with $\bar{Y}$ (the latter was calculated on the end of the last chapter to $\bar{Y} = 0.7 \times 10^{-4}\,\mathrm{m}$) shows that, only for Reynolds numbers near the critical value, the boundary layer thickness of the vapor flows can become comparable with the thickness of the film layer. In that case the ordinate y of the vapor may be measured from the interface. In general, however $\bar{Y}$ can be neglected against Y in the vapor. The boundary condition for the vapor flow at the interface can then be given simply by

$$y = 0 : u = v = 0. \tag{54}$$

Since for ducted flows, $\bar{Y}/R_0 \ll 1$, the boundary condition equation (54) is a very good approximation.

With equations (27) and (42), equation (32c) is reduced to

$$\dot{m} = \frac{\rho_r \rho_i v}{Y} \frac{\Delta_w}{Sc} \frac{C_e(T_w) - \bar{C}_e(T_w)}{C_e(T_i) - \bar{C}_e(T_i)} \left(\frac{\partial \Gamma}{\partial y}\right); \tag{55}$$

where Δ_w is an important parameter of concentration given by equation (27) at the wall temperature T_w and Sc is the Schmidt number of the vapor. The ratio of concentration differences at the wall and the interface

$$Q_{wi} = \frac{C_e(T_w) - \bar{C}_e(T_w)}{C_e(T_i) - \bar{C}_e(T_i)} \tag{56}$$

usually takes values close to unity as shown in Fig. 3.

7. CONDENSATION RATE AND INTERFACE TEMPERATURE

A comparison of the condensation rate expressed by the properties of the film flow, equation (37c), with that expressed by the properties of the vapor flow, equation (55) and (56), leads to

$$\bar{\delta}^2 \frac{\mathrm{d}\bar{\delta}}{\mathrm{d}x} = \frac{\rho_r}{\bar{\rho}_r} \frac{v}{\bar{V} Y} \frac{\Delta_w}{Sc} Q_{wi} \left(\frac{\partial \Gamma}{\partial y}\right)_i. \tag{57}$$

In usual cases of practical importance, the interface temperature is very close to the wall temperature, or in other words, θ_r, the reference value of θ_i, is quite small, $Q_{wi} \approx 1$. Estimate of the order of magnitude of the terms in equation (57) leads to

$$\frac{\rho_r}{\bar{\rho}_r} \frac{v}{\bar{V} Y} \frac{\Delta w}{Sc} = 1, \tag{58}$$

which gives the order of magnitude of $\bar{V}$. Using equation (40) for V and equation (51), the parameter K_t can be expressed by the fluid properties and the vapor-flow condition as follows:

$$K_t = \left(\frac{\rho_r}{\bar{\rho}_r} \frac{\Delta_w}{Sc}\right)^{4/3} \left(\frac{v}{\bar{v}}\right)^{2/3} \left(\frac{U^2}{gX}\right)^{1/3}. \tag{59}$$

The flow parameter in equation (59) U^2/gX is listed in Table 4 for each flow configuration.

By equations (40) and (59) with the value of U^2/gX in Table 4 and Table 3 the characteristic values of the condensate flow, $\bar{Y}$, $\bar{U}$ and $\bar{V}$ are obtainable (Table 5). The reference temperature at the interface is defined by equation (39) so that $\theta_i \approx \theta_r$ can be then given by

$$\theta_r = \frac{T_r - T_w}{T_0 - T_w} = \frac{\bar{P}r}{\bar{H}e} \left(\frac{v}{\bar{v}}\right)^{2/3} \left(\frac{\rho_r \Delta_w}{\bar{\rho}_r Sc}\right)^{4/3} \left(\frac{U^2}{gX}\right)^{1/3}. \tag{60}$$

Table 4. Flow parameter U^2/gX

	(a) Forced	(b) Natural
(1) Ducted	$U_0 v/(R_0^2 g)$	β
(2) Free	$U_0^2/(Lg)$	β

Table 5. Characteristic properties of condensate flow

		(a) Forced	(b) Natural
(1) Ducted	$\bar{Y}$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\mu}\frac{Re}{\bar{G}r}\right)^{1/3} R_0$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}\frac{Gr}{\bar{G}r}\right)^{1/3} R_0$
	$\bar{U}$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}\right)^{2/3}\left(\frac{\bar{G}r}{Re}\right)^{1/3}\frac{\bar{v}}{v}U_0$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}Gr\right)^{2/3}\bar{G}r^{1/3}\frac{\bar{v}}{R_0}$
	$\bar{V}$	$\frac{\Delta_w}{Sc}\frac{\rho_r}{\bar{\rho}_r}\frac{v}{R_0}$	$\frac{\Delta_w}{Sc}\frac{\rho_r}{\bar{\rho}_r}\frac{v}{R_0}$
		$Re = \frac{U_0 R_0}{v}\ \bar{G}r = \frac{gR_0^3}{\bar{v}^2}$	$Gr = \frac{\beta g R_0^3}{v^2}\ \bar{G}r = \frac{gR_0^3}{\bar{v}^2}$
(2) Free	$\bar{Y}$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}\frac{Re^{1/2}}{\bar{G}r}\right)^{1/3} L$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}\frac{Gr^{1/4}}{\bar{G}r}\right)^{1/3} L$
	$\bar{U}$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}\right)^{2/3}\left(\frac{\bar{G}r}{R_e^2}\right)^{1/3}\frac{\bar{v}}{v}U_0$	$\left(\frac{\Delta_w}{Sc}\frac{\mu}{\bar{\mu}}Gr^{1/4}\right)^{2/3}\bar{G}r^{1/3}\frac{\bar{v}}{L}$
	$\bar{V}$	$\frac{\Delta_w}{Sc}\frac{\rho_r}{\bar{\rho}_r}\frac{v}{L}Re^{1/2}$	$\frac{\Delta_w}{Sc}\frac{\rho_r}{\bar{\rho}_r}\frac{v}{L}Gr^{1/4}$
		$Re = \frac{UL}{v}\ \bar{G}r = \frac{gL^3}{\bar{v}^2}$	$Gr = \frac{\beta g L^3}{v^2}\ \bar{G}r = \frac{gL^3}{\bar{v}^2}$

With the same values as for the example following equation (41), $\bar{H}e = 0.05$; $\bar{P}r = 2$ and reasonable values $\bar{\rho}_r/\rho_r = 1.6 \times 10^3$; $v/\bar{v} = 80$; $\Delta_w/Sc = 1$, one gets:

$$\theta_r = 4 \times 10^{-2}\left(\frac{U^2}{gX}\right)^{1/3}.$$

This means that for forced flows at a plate of the length $L = 0.10$ m and $U_0 = 100\,\mathrm{m\,s^{-1}}$, θ_r will no longer be small. But this is flow near the critical Reynolds number, which requires further special consideration. As shown by Kotake [8], θ_r will be small in general.

Small values of θ_r mean that the condensation rate is dominated by the diffusion process in the vapor and as a consequence the condensation rate will essentially be smaller than that for one component flows with the same cooling condition. Substituting equation (58) into equation (57) with $Q_{wi} = 1$, $(\theta_r \ll 1)$ leads to

$$\bar{\delta}\frac{\mathrm{d}\delta}{\mathrm{d}x} = \left(\frac{\partial\Gamma}{\partial y}\right)_i. \qquad (61)$$

It implies that the film thickness δ for binary mixture condensation should be controlled by the concentration gradient at the interface. This is the most contrasting feature to that for one-component condensation. In the latter, the film thickness is governed by equation (37d) with equation (38b)

$$\delta^3\frac{\mathrm{d}\delta}{\mathrm{d}x} = \theta_i.$$

For free laminar flows along a flat plate, $(\partial\Gamma/\partial y)_i \sim \sqrt{x}$. From equation (61), $\bar{\delta} \sim \sqrt{x}$. When $(\partial\Gamma/\partial y) =$ constant, as in the case of fully developed ducted flows, $\bar{\delta} \sim \sqrt[3]{x}$. On the other hand for one component flow, $\theta_i = 1$, the above equation leads to $\delta \sim \sqrt[4]{x}$, which is the result of the Nusselt film theory.

Strictly speaking, the concentration gradient at the interface is a function of the x-coordinate, the Schmidt number Sc and the interfacial temperature θ_i. The latter involves the parameter $\theta_r = \bar{P}rK_l/\bar{H}e$. Thus, the gradient can be expressed in the form

$$\left(\frac{\partial\Gamma}{\partial y}\right)_i = f_{\dot{m}}(x, Sc, \bar{P}rK_l/\bar{H}_e).$$

In the case for small difference between wall and interface temperatures, $\theta_r \ll 1$, hence according to equation (39) $\bar{P}rK_l/\bar{H} \approx 0$, the concentration gradient can be evaluated at the wall temperature and at $y = 0$.

$$\left(\frac{\partial\Gamma}{\partial y}\right)_{y=0} = f_{\dot{m}}(x, Sc). \qquad (62)$$

With equation (55) and Table 3 one can obtain the following expressions of the condensation rate in $\dot{m}/\bar{\rho}\bar{V}$ for four different cases of flow configuration.

1a. *Ducted wall, forced convection*

$$\frac{\dot{m}R_0}{\rho_r v}\frac{Sc}{\Delta_w} = \frac{\dot{m}}{\rho_r U_0}Re\frac{Sc}{\Delta_w} = \rho_w f_{\dot{m}}(x, Sc);$$

$$Re = \frac{R_0 U_0}{\nu};$$

$$x = \frac{x'}{R_0 \cdot Re}, \qquad (63)$$

where the vapor density can be evaluated at wall temperature ($\theta = 0, \Gamma = 0$) according to equation (43):

$$\rho_w = 1 + \beta_t + \beta_c \qquad (64)$$

and can be quite different from one, Table 1.

1b. *Ducted wall, natural convection*

$$\frac{\dot{m}R_0}{\rho_r \nu}\frac{Sc}{\Delta_w} = \frac{\dot{m}}{\rho_r\sqrt{(\beta g R_0)}}Gr^{1/2}\frac{Sc}{\Delta_w}$$

$$= \rho_w f_{\dot{m}}(x, Sc);$$

$$Gr = \frac{\beta g R_0^3}{\nu^2};$$

$$x = \frac{x'}{R_0 Gr}. \qquad (65)$$

2a. *Free (plate), forced convection*

$$\frac{\dot{m}}{\rho_r \nu}L\sqrt{\left(\frac{\nu}{U_0 L}\right)}\frac{Sc}{\Delta_w} = \frac{\dot{m}}{\rho_r U_0}Re^{1/2}\frac{Sc}{\Delta_w}$$

$$= \rho_w f_{\dot{m}}(x, Sc);$$

$$Re = \frac{U_0 L}{\nu};$$

$$x = \frac{x'}{L}. \qquad (66)$$

2b. *Free (plate), natural convection*

$$\frac{\dot{m}}{\rho_r \nu}L\left(\frac{\nu^2}{\beta g L^3}\right)^{1/4}\frac{Sc}{\Delta_w} = \frac{\dot{m}}{\rho_r\sqrt{(\beta g L)}}Gr^{1/4}\frac{Sc}{\Delta_w}$$

$$= \rho_w f_{\dot{m}}(x, Sc);$$

$$Gr = \frac{\beta g L^3}{\nu^2};$$

$$x = \frac{x'}{L}. \qquad (67)$$

In the case of forced convective flows (1a) and (2a) the Reynolds number Re is involved in the factor of the condensation rate. For natural convection flows (1b) and (2b), the Reynolds number is replaced by the Grashof number Gr. The factor includes the parameters as a product, and they form a new parameter. The parameter, Δ_w/Sc is of the most importance. The function $f_{\dot{m}}$ depends on the boundary condition. It is therefore worthy of comparing the condensation rates of different binary mixtures under the same boundary conditions. Figure 4 shows the theoretical results of Kotake [8] for a pipe flow of constant cross-section at a velocity of $U_0 = 0.10\,\mathrm{m\,s^{-1}}$ and different cooling temperatures. In Fig. 4, Δ_w is not included in the coordinate while in Fig. 5 it is taken into account. In the latter, the curves are seen to be more coalesced. The discrepancies found in the figure arise from the difference of the development process of flow which is largely affected by the inlet condition. It is seen in the figure that some shift of the curves with respect to the x-parameter shows well correlated behavior.

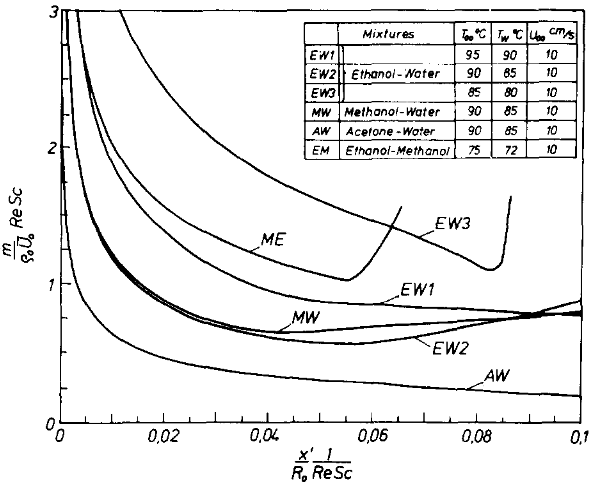


FIG. 4. Condensation rate of different binary mixtures for different wall temperatures.

8. CONCLUSIONS

Film condensation of binary vapor mixtures is studied on the basis of similarity consideration to obtain its general features. Film condensation of saturated one-component vapors is largely controlled by the removal mechanism of heat released at the liquid–vapor interface. On the contrary, film condensation of two-component vapor mixtures is mainly governed by the removal mechanism of mass caused by the difference of equilibrium concentrations at the interface. At the liquid–vapor interface of infinitesimal thickness where the mixture is in phase equilibrium, the liquid and vapor phases are subject to different equilibrium concentrations. The behavior of mass removal is characterized by the ratio Δ_w, equation (27), of the driving force of diffusion due to concentration difference between the interface and the bulk to the barrier force of the difference of equilibrium concentrations between the liquid and vapor phases.

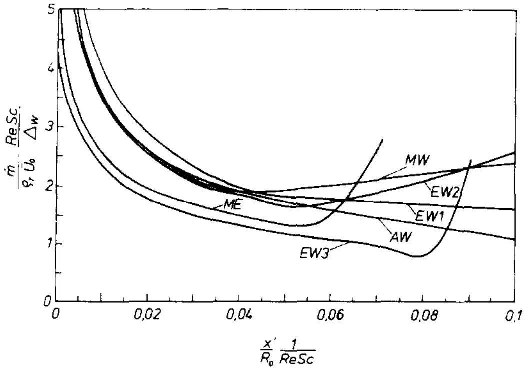


FIG. 5. Condensation rate of Fig. 4 divided by Δ_w, equation (27).

Although the equilibrium characteristics are inherent to the mixture, this mass removal factor can be regarded as a significant similarity parameter for film condensation of binary vapor mixture.

By examining the governing equations and the corresponding boundary conditions nondimensionalized with the reference values, the similarity conditions are obtained for binary mixture flows of film condensation and compared with those for the one-component system. Inspection of the interfacial temperature by introducing the reference temperature at the interface shows that the film thickness and the film velocity are to be of smaller values than those for the equivalent one-component film condensation. The nondimensionalized film thickness for binary mixtures is proportional to 0.5–0.33 of the power of the flow length, while that for one-component vapors is proportional to 0.25 of the power of the flowlength. The condensation rate as well as the characteristic properties of the flow such as the film thickness, the film velocity and the vapor velocity are expressed in terms of the similarity parameters for forced flows and natural convection flows in ducted walls and along a plate. The obtained similarity is compared with a numerical result to show well correlated behavior, although further comparisons with experimental results are required.

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PARAMETRES DE CONDENSATION EN FILM D'UN MELANGE BINAIRE

Résumé—En ce qui concerne l'écoulement du film liquide, la condensation en film d'un mélange binaire est semblable en de nombreux points à la théorie de Nusselt. La chaleur est évacuée par le film seulement. De plus, prend place un mécanisme de diffusion dans l'écoulement saturé gazeux. Il cause le transport du composant le plus volatil du mélange dans la direction opposée à l'interface. Malgré le grand nombre de groupements sans dimension, seuls quelques paramètres sont essentiels pour caractériser l'écoulement. Des mélanges binaires très différents sont comparés. Dans cette étude, on ne considère que des écoulements laminaires sur une plaque plane et entre des parois parallèles, mais la discussion des paramètres essentiels doit être d'une portée générale.

PARAMETER BINÄRER FILMKONDENSATIONSGEMISCHE

Zusammenfassung—Die Filmkondensation binärer Gemische zeigt betreffs des Flüssigkeitsfilmes starke Ähnlichkeit mit der Nusseltschen Theorie. Durch den Film allein erfolgt die Wärmeabfuhr. Dazu tritt aber noch ein Diffussionsprozeß in dem als strömend angenommenen gesättigten Gas. Er muß den Abtransport der flüchtigeren Gemischkomponente von der Phasengrenze besorgen. Trotz der Fülle der Kennzahlen sind nur einige wenige Parameter für den Vorgang wesentlich, so daß Prozesse sehr unterschiedlicher binärer Gemische miteinander vergleichbar werden. In der Arbeit werden nur die laminaren ebenen Strömungen an einer Platte und zwischen parallelen Wänden untersucht, doch dürfte den wesentlichen Parametern weit allgemeinere Bedeutung zukommen.

ПАРАМЕТРЫ ПЛЕНОЧНОЙ КОНДЕНСАЦИИ БИНАРНОЙ СМЕСИ

Аннотация — Течение жидкой пленки при пленочной конденсации бинарной смеси можно достаточно точно описать с помощью теории Нуссельта. Отбор тепла осуществляется только пленкой. Кроме того, в потоке насыщенных паров происходит диффузия, вызывающая перенос более летучей компоненты смеси в сторону от границы раздела. Несмотря на наличие большого числа безразмерных параметров, только некоторые из них могут использоваться для характеристики потока. С их помощью можно проводить сравнительный анализ процессов с участием самых различных бинарных смесей. В данной работе рассматриваются только ламинарные течения на плоской пластине и между параллельными стенками, однако проведенный анализ важнейших параметров процесса может представлять более широкий интерес.